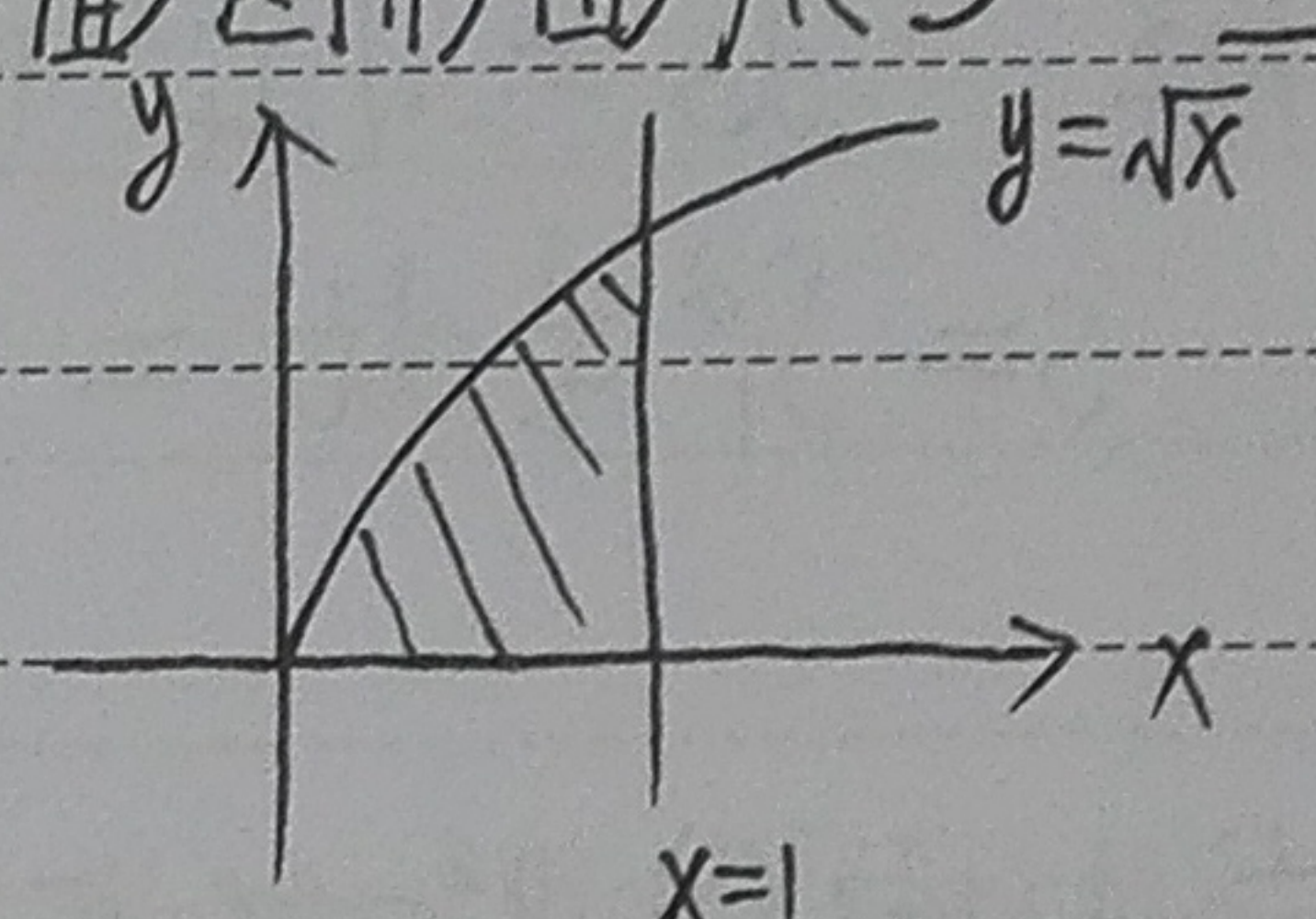
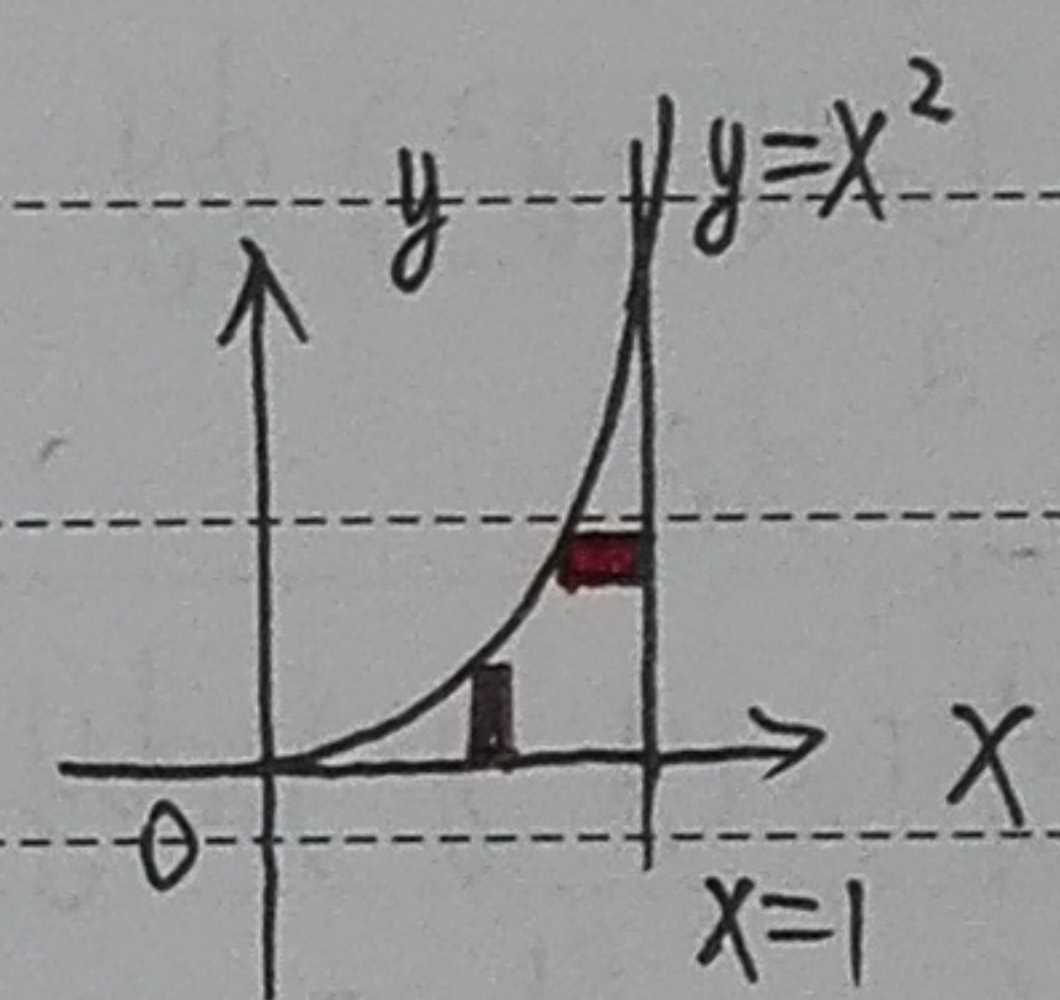


华东交通大学考试专用答题纸 (适用于匿名改卷)

课程: 6.5 定积分在几何上的应用

题号	一	二	三	四	五	六	七	八	九	十	总分
得分											
阅卷人											

考生注意: 答题时应注明题号, 并按题号顺序答题。

题号	答题内容
P131	廖国勇
一 1	由曲线 $y=\sqrt{x}$ 与直线 $x=1, y=0$ 围成的平面图形面积 $S=$ _____ 解: $S = \int_0^1 \sqrt{x} dx$ $= \frac{2}{3} x^{\frac{3}{2}} \Big _0^1 = \frac{2}{3}$ 或者 $S = \int_0^1 (1-y^2) dy$ $= (y - \frac{1}{3} y^3) \Big _0^1 = 1 - \frac{1}{3} = \frac{2}{3}$ 
2	由曲线 $y=x^2$ 与直线 $x=1, y=0$ 围成的平面图形绕 x 轴旋转一周所形成的旋转体体积 $V=$ _____ 解: $dV = \pi y^2 dx = \pi x^4 dx$ $V = \int_0^1 \pi x^4 dx = \frac{\pi}{5} x^5 \Big _0^1 = \frac{\pi}{5}$ $dV = \pi [(x^2)^2 + 1] dy = \pi (1+y)$ $V = \int_0^1 \pi (1+y) dy =$ $dV = 2\pi y \cdot (1-x) dx = 2\pi x^2 (1-x) dx = (2\pi x^2 - 2\pi x^3) dx$ $= 2\pi y (1-\sqrt{y}) dy = (2\pi y - 2\pi y^{\frac{3}{2}}) dy$ $V = \int_0^1 (2\pi y - 2\pi y^{\frac{3}{2}}) dy = 2\pi (\frac{y^2}{2} - \frac{2}{5} y^{\frac{5}{2}}) \Big _0^1$ $= 2\pi (\frac{1}{2} - \frac{2}{5}) = \frac{\pi}{5}$ 

题号

P31

答题内容

廖国勇

一 3 曲线 $y = \frac{2}{3}x^{\frac{3}{2}}$ 在区间 $[0, 1]$ 上的长度 $S =$ —

$$\text{解: } ds = \sqrt{1+y'^2} dx = \sqrt{1+(6x)^2} dx = \sqrt{1+x} dx$$

$$S = \int_0^1 \sqrt{1+x} dx = \int_0^1 (1+x)^{\frac{1}{2}} d(1+x)$$

$$= \frac{2}{3} (1+x)^{\frac{3}{2}} \Big|_0^1 = \frac{2}{3} (2\sqrt{2} - 1)$$

4 曲线 $\begin{cases} x = \int_1^t \frac{\cos u}{u} du \\ y = \int_1^t \frac{\sin u}{u} du \end{cases}$ 在区间 $[1, 2]$ 上的长度 $S =$ —

$$\text{解: } ds = \sqrt{x_t'^2 + y_t'^2} dt = \sqrt{\left(\frac{\cos t}{t}\right)^2 + \left(\frac{\sin t}{t}\right)^2} dt = \frac{1}{t} dt$$

$$S = \int_1^2 \frac{1}{t} dt = \ln t \Big|_1^2 = \ln 2$$

二 1 求由曲线 $y^2 = x$ 与直线 $x - y = 2$ 围成的平面图形面积

$$\text{解: } \begin{cases} y^2 = x \\ x - y = 2 \end{cases} \Rightarrow \begin{cases} x = 4 \\ y = 2 \end{cases} \text{ 或 } \begin{cases} x = 1 \\ y = -1 \end{cases}$$

$$dA = [(2+y) - y^2] dy$$

$$= (2+y-y^2) dy$$

$$A = \int_{-1}^2 dA$$

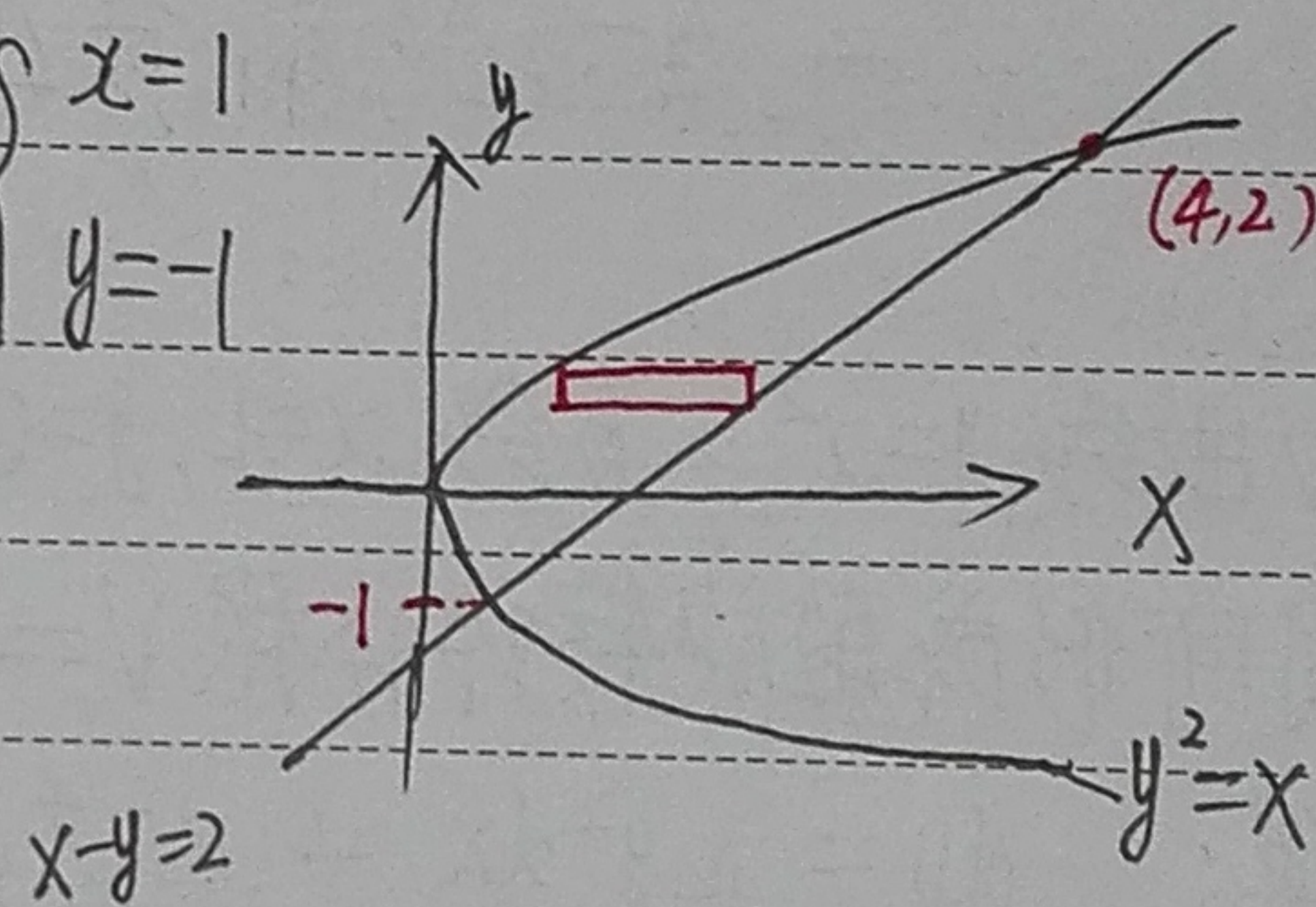
$$= \int_{-1}^2 (2+y-y^2) dy$$

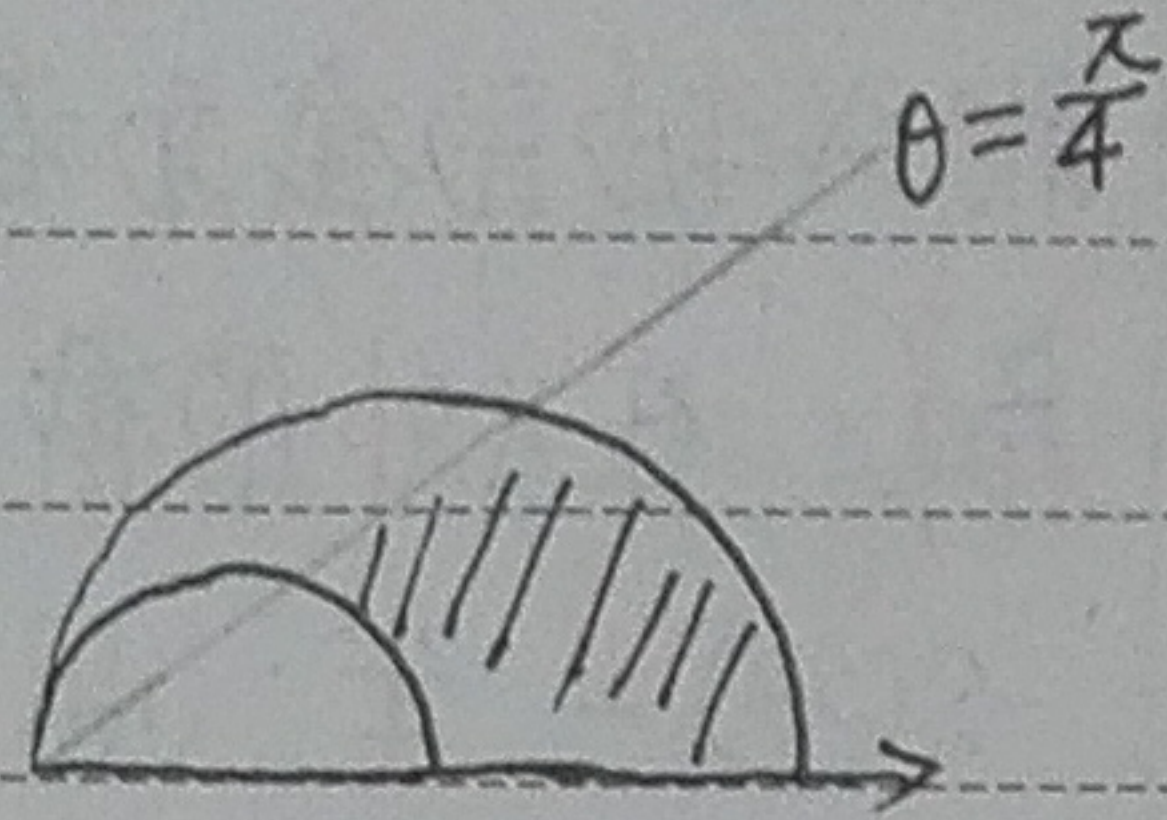
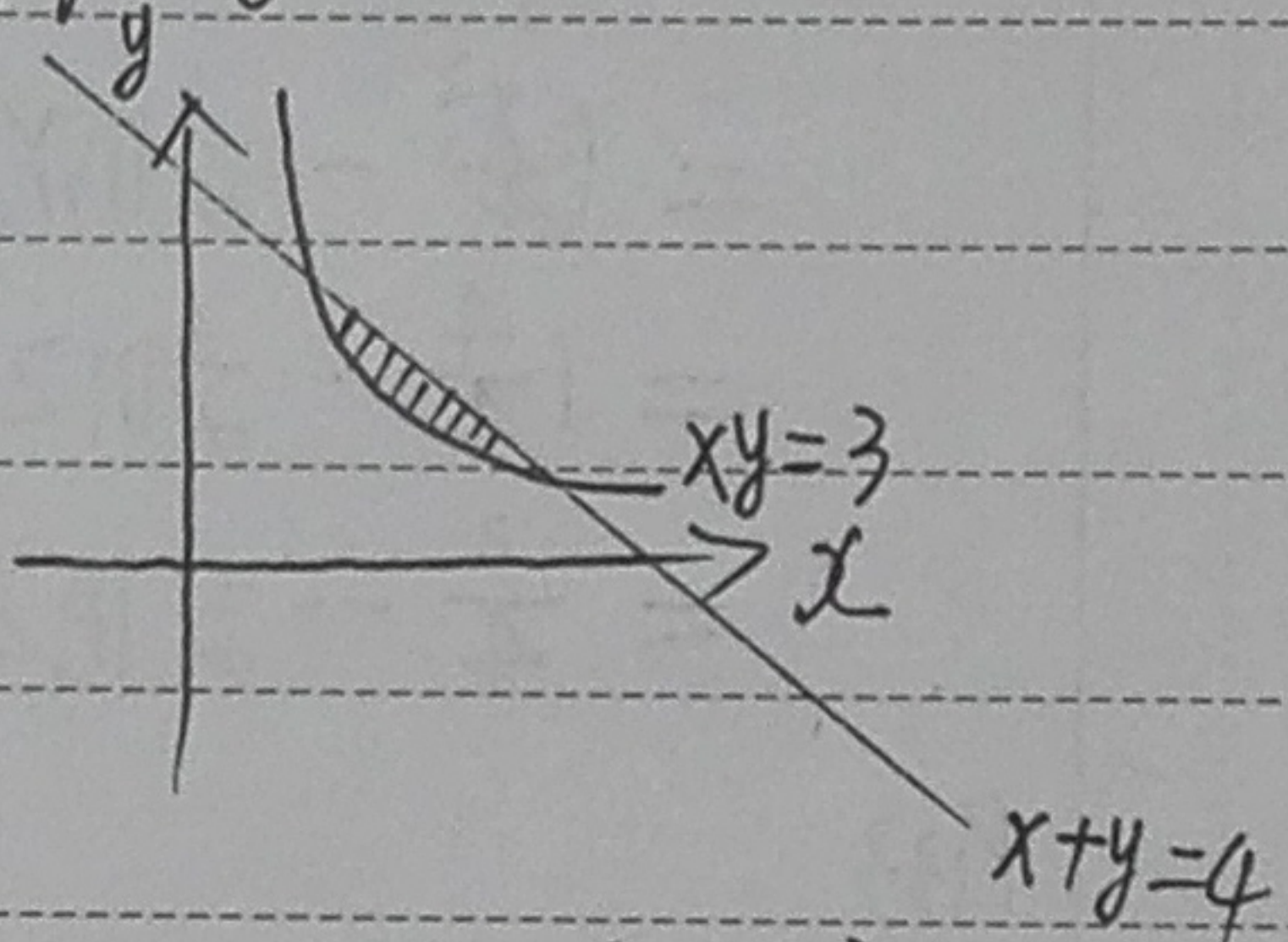
$$= \left(2y + \frac{1}{2}y^2 - \frac{1}{3}y^3 \right) \Big|_{-1}^2$$

$$= \left(4 + 2 - \frac{8}{3} \right) - \left(-2 + \frac{1}{2} - \frac{1}{3} \right)$$

$$= \frac{10}{3} + \left(\frac{1}{6} \right) \frac{7}{6} = \frac{27}{6} = \frac{9}{2}$$

$$= \frac{10}{3} + \frac{13}{6} = \frac{33}{6} = \frac{11}{2}$$



题号	答题内容	廖国勇
二 2	求由圆周 $r = \cos\theta$, $r = 2\cos\theta$ 围成且介于 $\theta = 0$ 与 $\theta = \frac{\pi}{4}$ 之间的平面图形面积. 解: $dA = \frac{1}{2} [(2\cos\theta)^2 - (\cos\theta)^2] d\theta$ $= \frac{3}{2} \cos^2\theta d\theta$ $A = \int_0^{\frac{\pi}{4}} \frac{3}{2} \cos^2\theta d\theta$ $= \frac{3}{4} \int_0^{\frac{\pi}{4}} (1 + \cos 2\theta) d\theta$ $= \frac{3}{4} \left(\theta + \frac{1}{2} \sin 2\theta \right) \Big _0^{\frac{\pi}{4}}$ $= \frac{3}{4} \left(\frac{\pi}{4} + \frac{1}{2} \cdot 1 \right)$ $= \frac{3}{16}\pi + \frac{3}{8}$	
3	求由曲线 $xy = 3$ 及直线 $x + y = 4$ 围成的平面图形绕 x 轴及 y 轴旋转一周所形成的旋转体体积. 解 $\begin{cases} xy = 3 \\ x + y = 4 \end{cases} \Rightarrow \begin{cases} x = 1 \\ y = 3 \end{cases} \text{ 或 } \begin{cases} x = 3 \\ y = 1 \end{cases}$ $dV_x = \pi [(4-x)^2 - (\frac{3}{x})^2] dx$ $= \pi (16 - 8x + x^2 - \frac{9}{x^2}) dx$ $V_x = \int_1^3 \pi (16 - 8x + x^2 - \frac{9}{x^2}) dx = \pi (16x - 4x^2 + \frac{1}{3}x^3 + \frac{9}{x}) \Big _1^3$ $= \pi (48 - 36 + 9 + 3) - \pi (16 - 4 + \frac{1}{3} + 9) = \frac{8}{3}\pi$ $dV_y = \pi [(4-y)^2 - (\frac{3}{y})^2] dy = \pi (16 - 8y + y^2 - \frac{9}{y^2}) dy$ $V_y = \int_1^3 \pi (16 - 8y + y^2 - \frac{9}{y^2}) dy$ $= V_x$ $= \frac{8}{3}\pi$	

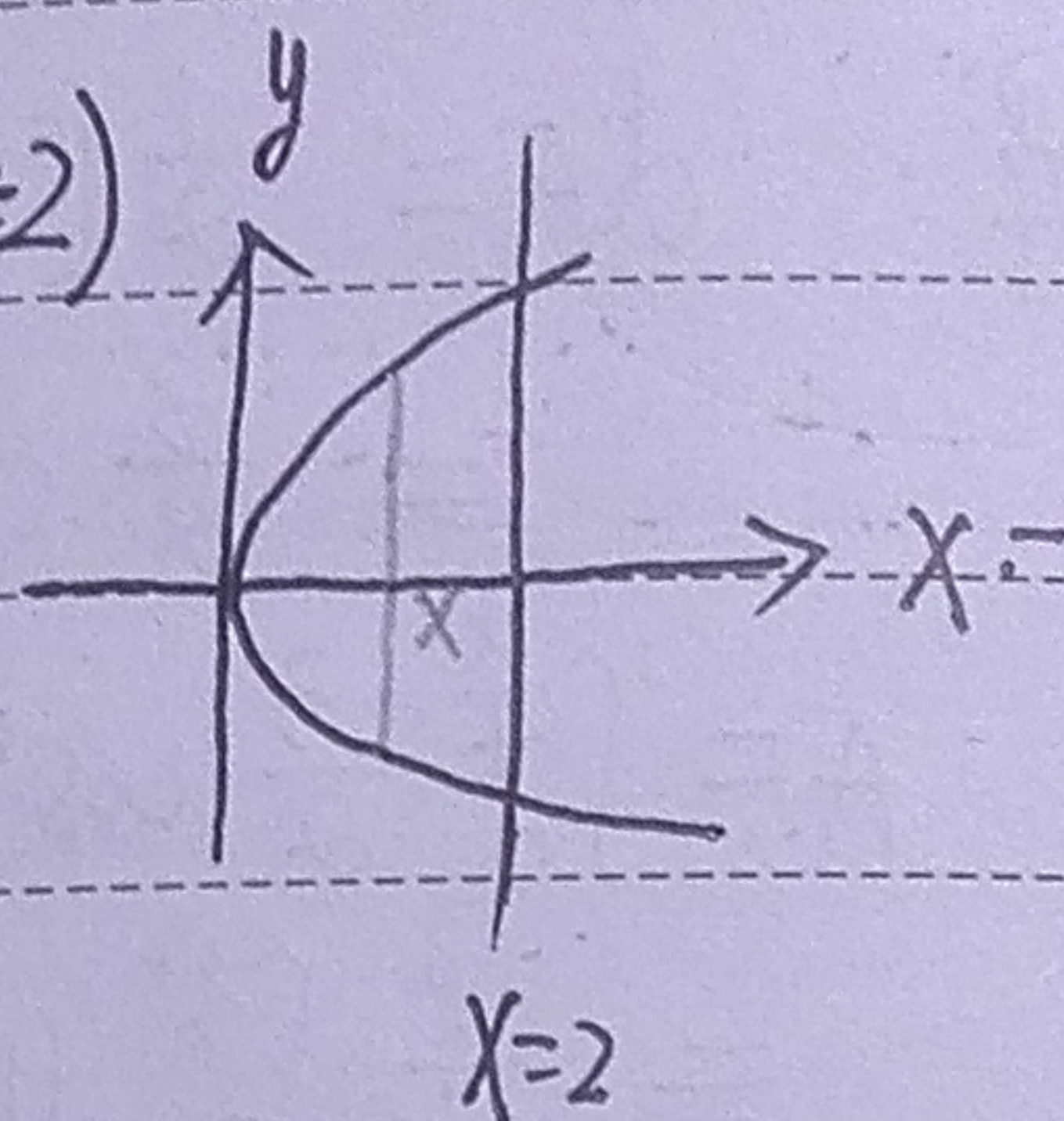
- 二 4 一立体以抛物线 $y^2 = 2x$ 与直线 $x = 2$ 围成的图形为底, 而垂直于抛物线轴的截面都是等边三角形. 求其体积.

解: 在 x 处的截面三角形面积 $(0 \leq x \leq 2)$

$$S(x) = \frac{\sqrt{3}}{4} \cdot (\sqrt{2x} + \sqrt{2x})^2 = 2\sqrt{3}x$$

$$V = \int_0^2 S(x) dx$$

$$= \int_0^2 2\sqrt{3}x dx = \sqrt{3}x^2 \Big|_0^2 = 4\sqrt{3}$$



- 5 求曲线 $y = \frac{x^2}{2} - \frac{1}{2}\ln x$ 在 $[1, 2]$ 上的长度

解: $ds = \sqrt{1 + y'^2} dx$

$$= \sqrt{1 + \left(\frac{x}{2} - \frac{1}{2x}\right)^2} dx$$

$$= \left(\frac{x}{2} + \frac{1}{2x}\right) dx$$

$$s = \int_1^2 ds = \int_1^2 \left(\frac{x}{2} + \frac{1}{2x}\right) dx$$

$$= \left(\frac{x^2}{4} + \frac{1}{2}\ln x\right) \Big|_1^2$$

$$= \left(\frac{4}{4} + \frac{1}{2}\ln 2\right) - \left(\frac{1}{4} + \frac{1}{2}\ln 1\right)$$

$$= \frac{3}{4} + \frac{1}{2}\ln 2$$

P133

- 6 求曲线 $r = 2(1 + \cos \theta)$ (图6-1) 的长度

解: $ds = \sqrt{r^2 + r'^2} d\theta$

$$= \sqrt{4(1 + \cos \theta)^2 + (-2\sin \theta)^2} d\theta$$

$$= 4\left|\cos \frac{\theta}{2}\right| d\theta$$

$$s = \int_0^{2\pi} ds = 2 \int_0^{\pi} ds$$

$$= 2 \int_0^{\pi} 4 \cos \frac{\theta}{2} d\theta = 16 \sin \frac{\theta}{2} \Big|_0^{\pi}$$

$$= 16$$

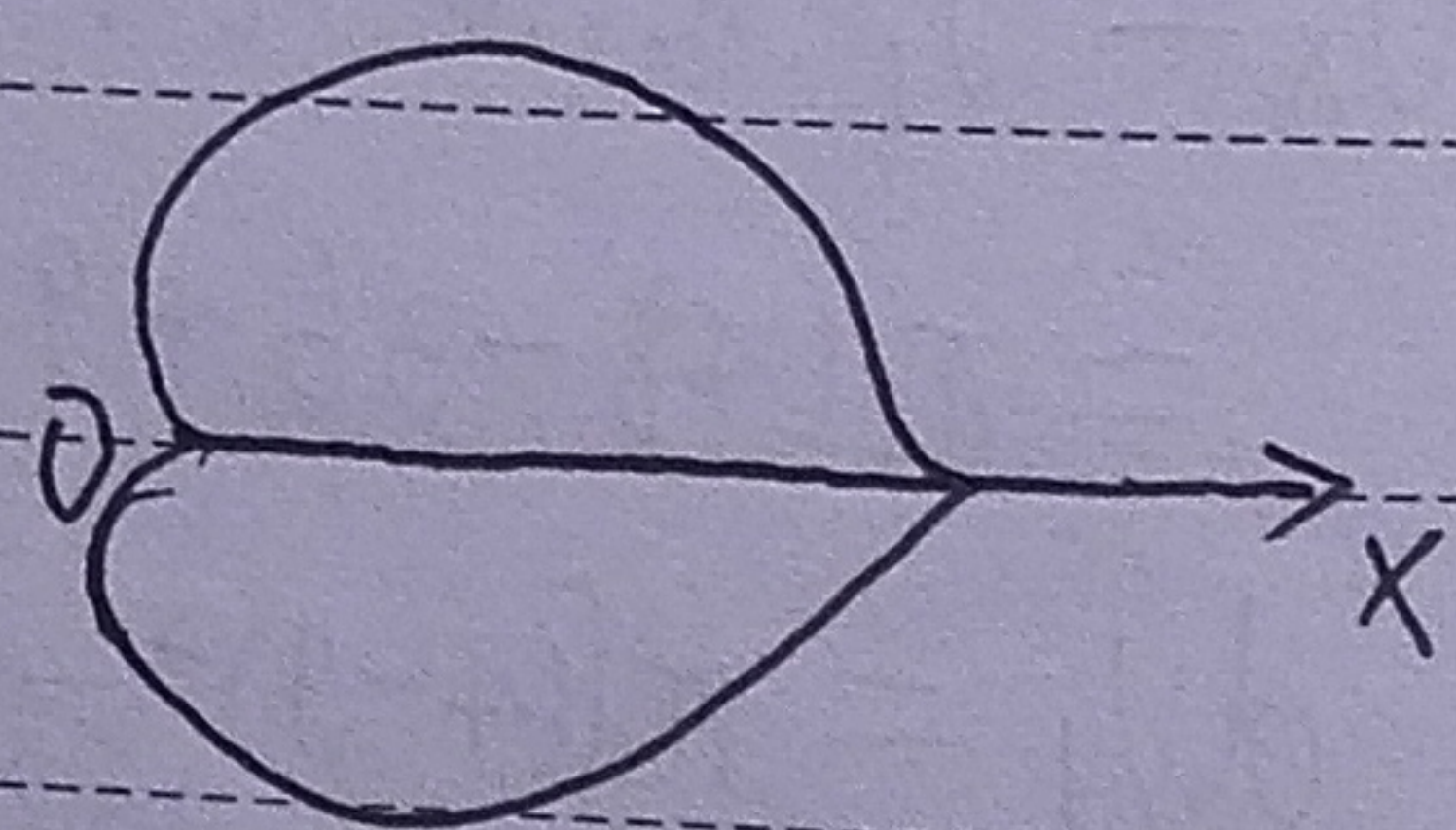


图6-1

题号

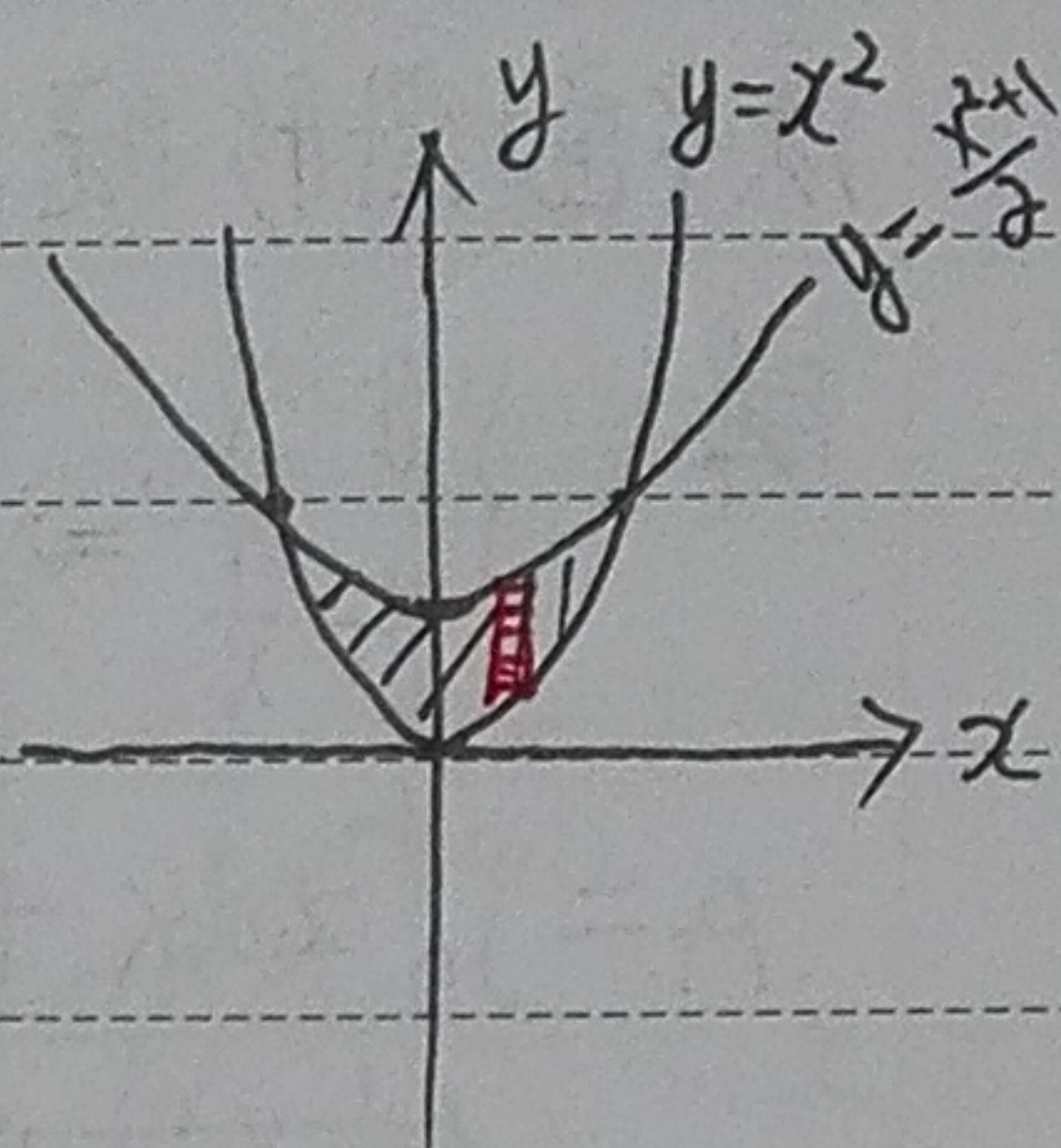
P133

答 题 内 容

廖国勇

7 求由曲线 $y=x^2$, $y=\frac{x^2+1}{2}$ 围成的平面图形面积及该平面图形绕 y 轴旋转一周所形成的旋转体体积.

$$\text{解: } \begin{cases} y=x^2 \\ y=\frac{x^2+1}{2} \end{cases} \Rightarrow \begin{cases} x=-1 \\ y=1 \end{cases} \text{ 或 } \begin{cases} x=1 \\ y=1 \end{cases}$$



$$dA = \left(\frac{x^2+1}{2} - x^2 \right) dx$$

$$= \frac{1-x^2}{2} dx$$

$$A = \int_{-1}^1 \frac{1-x^2}{2} dx = 2 \int_0^1 \frac{1-x^2}{2} dx$$

$$= 2 \cdot \left(\frac{1}{2}x - \frac{1}{6}x^3 \right) \Big|_0^1$$

$$= 2 \left(\frac{1}{2} - \frac{1}{6} \right) - \frac{1}{2}(0-0)$$

$$= \frac{2}{3}$$

$$dV_y = 2\pi x \left(\frac{x^2+1}{2} - x^2 \right) dx$$

$$= \pi x (x^2+1-2x^2) dx = (\pi x - \pi x^3) dx$$

$$V_y = \int_0^1 (\pi x - \pi x^3) dx$$

$$= \left(\frac{\pi}{2}x^2 - \frac{\pi}{4}x^4 \right) \Big|_0^1$$

$$= \left(\frac{\pi}{2} - \frac{\pi}{4} \right) - (0-0)$$

$$= \frac{\pi}{4}$$

题号

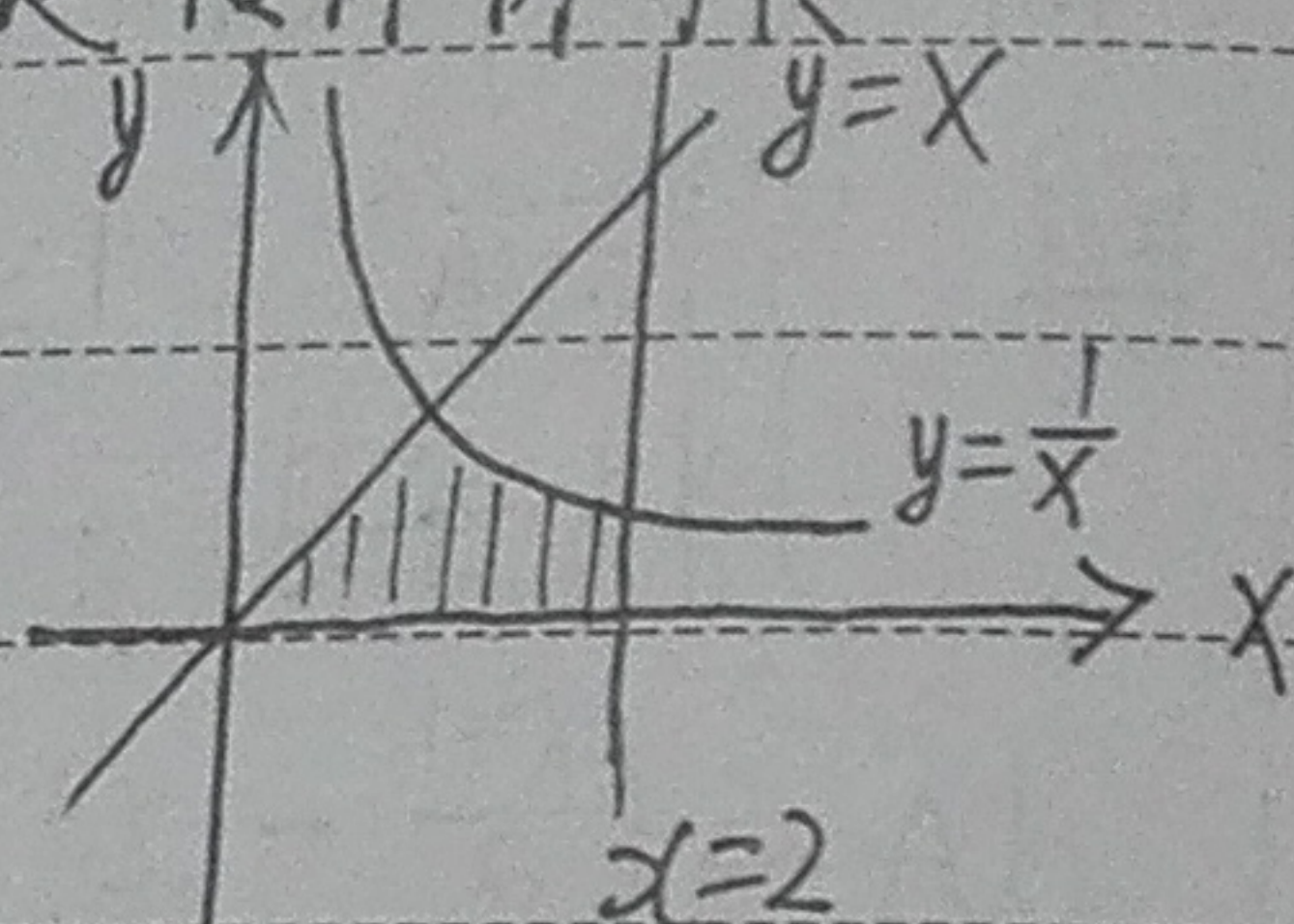
P133

答题内容

廖国勇

8 求由曲线 $y=x$, $y=\frac{1}{x}$, $x=2$ 及 $y=0$ 围成的平面图形面积及该平面图形绕 x 轴旋转一周所形成的旋转体体积

$$\text{解: } \begin{cases} y=x \\ y=\frac{1}{x} \end{cases} \Rightarrow \begin{cases} x=1 \\ y=1 \end{cases}$$



$$A = \int_0^1 x dx + \int_1^2 \frac{1}{x} dx = \frac{1}{2} + \ln 2$$

$$= \left(\frac{1}{2} x^2 + \ln x \right) \Big|_0^2$$

$$V_x = \int_0^1 \pi x^2 dx + \int_1^2 \pi \left(\frac{1}{x} \right)^2 dx$$

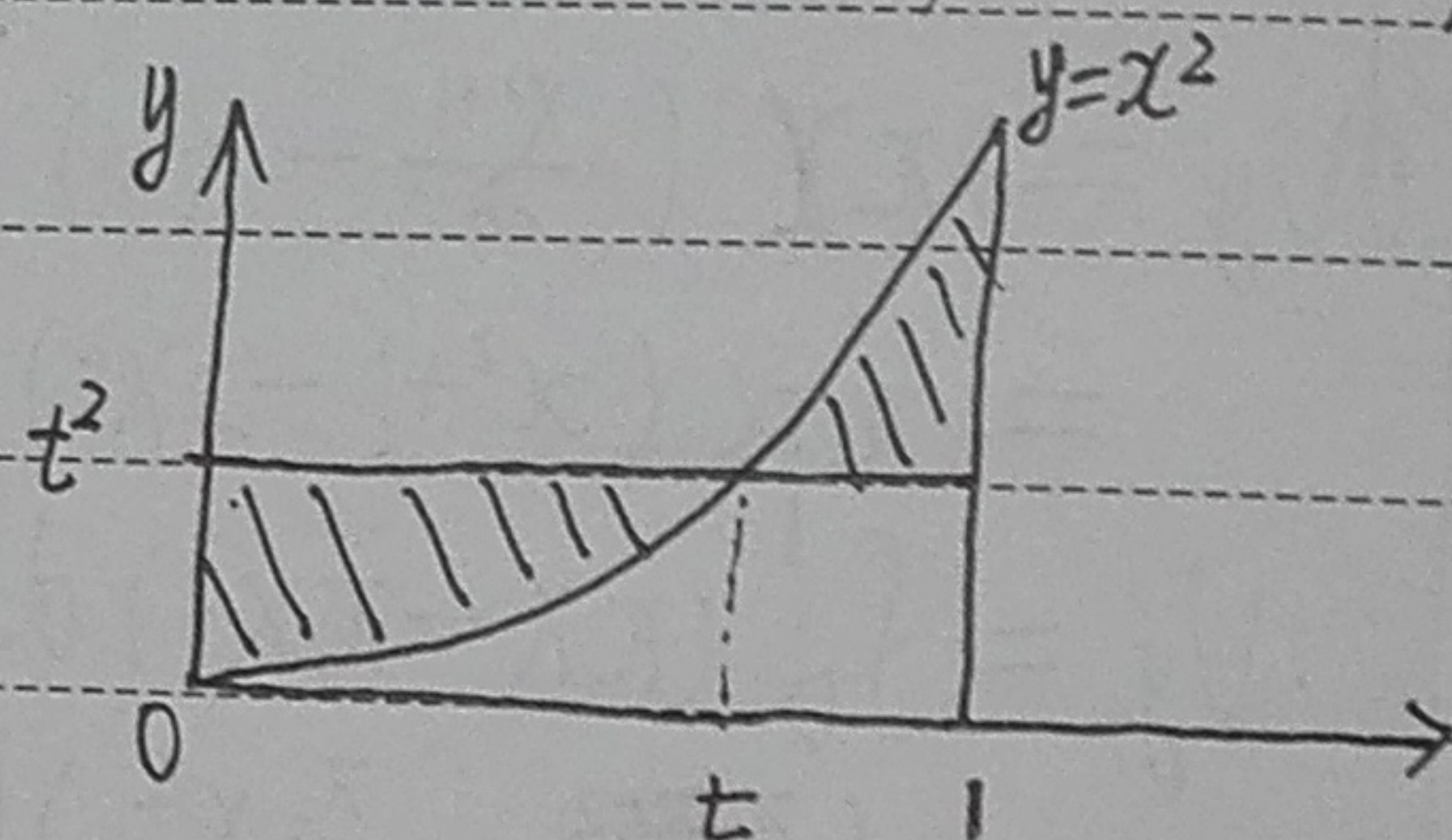
$$= \frac{\pi}{3} + \left(-\frac{\pi}{x} \right) \Big|_1^2 = \frac{\pi}{3} - \frac{\pi}{2} + \pi = \frac{5}{6} \pi$$

9 设 $y=x^2$ 定义在闭区间 $[0,1]$ 上, t 是 $[0,1]$ 上任一点, 当 t 为何值时, 图中阴影部分面积最小

$$\text{解: } A(t) = \int_0^t (t^2 - x^2) dx$$

$$+ \int_t^1 (x^2 - t^2) dx$$

$$= \frac{1}{3} t^3 - \frac{1}{3}$$



$$= (t^3 - \frac{1}{3} t^3) + \left(\frac{x^3}{3} - t^2 x \right) \Big|_t^1$$

$$= \frac{2}{3} t^3 + \left(\frac{1}{3} - t^2 \right) + \frac{2}{3} t^3$$

$$= \frac{4}{3} t^3 - t^2 + \frac{1}{3}$$

$$0 \leq t \leq 1$$

$$A'(t) = 4t^2 - 2t \quad \text{令 } A'(t) = 0 \Rightarrow t=0 \text{ 或 } t=\frac{1}{2}$$

$A''(t) = 8t - 2$, $A''(\frac{1}{2}) > 0$, $t=\frac{1}{2}$ 为内部唯一驻点
且 $f(\frac{1}{2})$ 取极小值, 故 $f(\frac{1}{2})$ 为 $A(t)$ 在 $[0,1]$ 上的最小值,
所以 $t=\frac{1}{2}$ 时, 图中阴影部分面积最小。

6.6

题号

P134

答 题 内 容

廖国勇

二 10

经过坐标原点作曲线 $y = \ln x$ 的切线, 该曲线 $y = \ln x$ 与切线及 x 轴围成的平面图形为 D . 求:

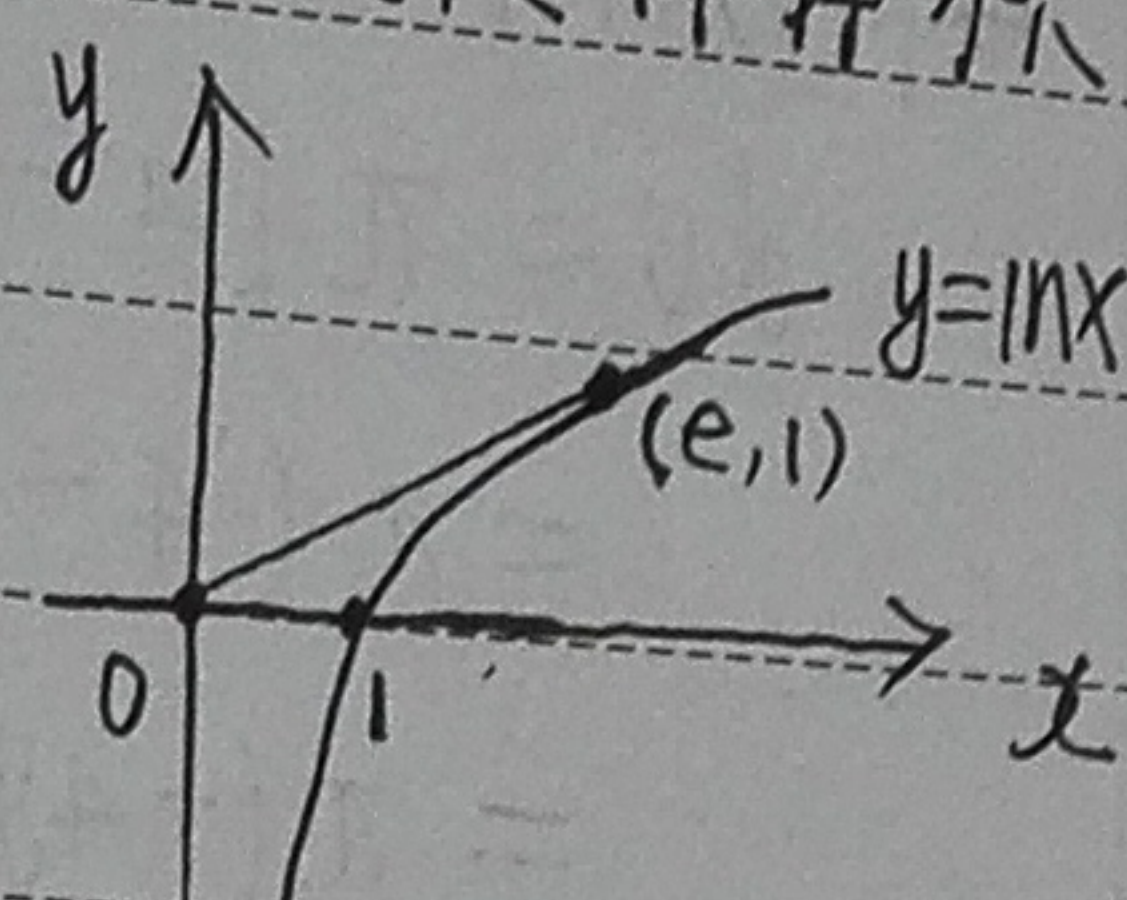
(1) D 的面积 (2) D 绕 y 轴旋转一周所形成的旋转体体积.

解: 设切点坐标为 $(x_0, \ln x_0)$, 则

由切线的几何意义

$$k_{\text{切}} = y'(x_0) = \frac{\ln x_0 - 0}{x_0 - 0}$$

$$\Rightarrow \frac{1}{x_0} = \frac{\ln x_0}{x_0} \Rightarrow x_0 = e \quad \text{切线方程 } y = \frac{1}{e}x$$



(1) 面积元素 $dA = (e^y - e^{\frac{1}{2}y}) dy$

$$A = \int_0^1 dA$$

$$= \int_0^1 (e^y - e^{\frac{1}{2}y}) dy$$

$$= (e^y - \frac{1}{2}e^{\frac{1}{2}y}) \Big|_0^1$$

$$= (e - \frac{1}{2}e) - (1 - 0) = \frac{e}{2} - 1$$

(2) 体积元素 $dV_y = \pi[(e^y)^2 - (e^{\frac{1}{2}y})^2] dy = \pi(e^{2y} - e^y) dy$

$$V_y = \int_0^1 dV_y$$

$$= \int_0^1 \pi(e^{2y} - e^y) dy$$

$$= \pi(\frac{1}{2}e^{2y} - e^y) \Big|_0^1$$

$$= \pi(\frac{e^2}{2} - e) - \pi(\frac{1}{2} - 1)$$

$$= \frac{\pi}{6}e^2 - \frac{\pi}{2}$$

$$= \frac{\pi}{6}(e^2 - 3)$$

题号

P134

答 题 内 容

廖国勇

11 求由曲线 $x=2\sqrt{y}$ 及直线 $x=0, y=1$ 围成的平面图形绕 $x=2$ 旋转一周所形成的旋转体体积

解: 设体积元素为 dV , 则

$$dV = \pi [2^2 - (2 - 2\sqrt{y})^2] dy$$

$$= \pi (4 - 4 - 4y + 8\sqrt{y}) dy$$

$$= \pi (-4y + 8\sqrt{y}) dy$$

$$V = \int_0^1 dV$$

$$= \int_0^1 \pi (-4y + 8\sqrt{y}) dy$$

$$= \pi (-2y^2 + \frac{16}{3} y^{\frac{3}{2}}) \Big|_0^1$$

$$= \pi (-2 + \frac{16}{3}) - \pi (0 + 0)$$

$$= \frac{16}{3} \pi - 2\pi$$

$$= \frac{10}{3} \pi$$

